MA614 HW8a-08 (due M. April 7, 2008)

1. Let λ, $\mu \in \Lambda(A)$ with $\lambda \neq \mu$, prove that any left eigenvector of A corresponding to μ is orthogonal to any right eigenvector of A corresponding to λ.

2. Let A be real, and let $\lambda = \alpha + i\beta$ be a complex eigenvalue of A with eigenvector $x + iy$, show that the space spanned by x and y is an invariant subspace of A.

3. If n by n matrix A has eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, show that the following statements are equivalent:

   (a) A is normal ($A^H A = AA^H$);

   (b) A is unitarily diagonalizable, and

   (c) $\sum_{i,j=1}^{n} |a_{ij}|^2 = \sum_{i=1}^{n} |\lambda_i|^2$.

4. A matrix $A \in \mathbf{C}^{n\times n}$ is said to be *skew Hermitian* if $A^H = -A$. Prove that

   (a) the eigenvalues of a skew Hermitian are purely imaginary.

   (b) $I - A$ is nonsingular.

   (c) $C = (I - A)^{-1}(I + A)$ is unitary.

   C is called Cayley transform of A.